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Estimation of Soil Strength Properties for Critical Rooting Conditions

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ABSTRACT

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Methods to aid in the large-scale testing and characterization of Coastal Plain soils based on their susceptibility to root-limiting strength problems were developed and analyzed. They were basically regression equations modeled after a Taylor series expansion. The equations relate changes of soil strength, bulk density and soil water content between field and "critical rooting conditions". Once equations were developed from a data set of 426 laboratory samples, critical rooting bulk density was predicted for a separate set of laboratory and field samples. All laboratory samples and appropriate field samples were equilibrated at −100 kPa soil-water potential. Soils used were sandy Ultisols, which may limit the scope of equations.

In many cases, changes in the water contents were not a significant factor in the prediction of soil strength. This may be a reflection of the limited capabilities of the equations, the uniform equilibration of soil-water potential of the soils, or the fact that the slope of the strength vs. bulk density curve is independent of water content over the range of samples considered. Nevertheless, it does simplify the equations and may suggest that a series of several equations for different soil types would be better than a single equation that requires soil-water content.

INTRODUCTION

Proper rooting is essential for plant growth. However, in the Southeastern Coastal Plain, subsoils with high bulk density and low water-holding capacity impede root development (Campbell et al., 1974; Stitt et al, 1982; Box and Langdale, 1984). Root growth ceases after soil strength reaches some critical value that is influenced by texture and plant species (Gerard et al., 1982). It is taken here to be 2 MPa resistance to the passage of a 5-mm diameter, flat-tipped penetrometer (Taylor et al., 1966; Camp and Lund, 1968). Growth is reduced at a lesser strength.

It would be useful to characterize soils susceptible to strength problems so that proper tillage management and rooting could be anticipated. One might use existing data such as texture (Daddo and Warrington, 1983; Spivey et al., 1986) or require the measurement of soil strength at some standard condition, such as cores brought to equilibrium at a specific soil-water potential and compacted by a common methodology. For the second method, the soil strength would be needed at critical rooting conditions and perhaps at some lesser strengths which reduce but do not stop growth. However, it would be difficult to attain critical rooting conditions on a routine basis. Furthermore, it would be time-consuming to develop a regression that would give such conditions for each sample.

The following procedure provides a way to estimate soil bulk density of a soil sample at -100 kPa of soil-water potential and 2 MPa of strength by knowing soil characteristics at some similar strength value. Basic assumptions in this are: (1) it is easier and more accurate to measure a change in bulk density between some known point and the critical rooting bulk density (CRBD)[1] than to measure the critical conditions directly; (2) water content at critical conditions can be approximated from critical rooting bulk density or by some other means if it is needed.

An advantage of this approach is that it does not demand that the same regression equation be valid for all soils, but rather that all soils fit into a family of equations with a uniform slope of strength vs. bulk density (Gupta and Larson, 1982; Saini et al., 1984).

METHODS AND MATERIALS

Sampling

Soils for this study were chosen to represent the texture and organic matter content of the Paleudults of the Southeastern Coastal Plains. They have sandy-textured, weakly-structured plow layers underlain by root-restricting, compacted layers. Other soils were included in the study to provide a range of surface soil texture.

Soil samples were collected at 17 locations, and taxonomically documented by the Soil Conservation Service (Spivey et al., 1986). At each site, a loose, moist sample of the surface soil layer, weighing approximately 7 kg, was collected for laboratory analysis.

Soil samples for laboratory analysis were crushed by hand and passed through a 2-mm sieve to remove roots, leaves and pebbles. When necessary, water was

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[1]Some definitions for the terminology used are: CRBD = critical rooting bulk density (bulk density at 2 MPa soil strength and -100 kPa soil-water potential); CRWC = critical rooting water content (soil-water content at CRBD); WCBD = water-consolidated bulk density (bulk density of samples consolidated by saturation and drying to -100 kPa soil-water potential).

added by sprinkling to bring each sample to a water content equivalent of approximately -10 to -30 kPa soil-water potential. The moist soil was thoroughly mixed by rolling on a polyethylene sheet before removal of subsamples for analysis.

Compression

Soil samples were compacted by mechanical force and by water consolidation. Soil for mechanically-compacted samples was weighed and compressed into a known volume to give a desired dry bulk density. Soil cores were prepared for a range of bulk densities at intervals of approximately 0.05 Mg m^{-3}. The range was determined by the ease of compaction.

Compacted cores were moistened by placing them on a coarse, wet sand bed where the water level in the sand was maintained about 13 mm below the sand surface. Samples were then placed on standard ceramic pressure plates and equilibrated at -100 kPa soil-water potential.

After soil strength measurements were made (as described later), the test cores were oven-dried and bulk density was determined. Final bulk density was usually slightly different from the target value because of the non-uniform compaction of the one-directional mechanical compression. Differences between actual and targeted bulk densities averaged 0.026 Mg m^{-3} with a standard deviation of 0.015 Mg m^{-3}.

Soil cores were also compacted using a water consolidation technique and analyzed separately. In this technique, 76-mm-diameter sleeves were placed on a wet ceramic pressure plate which was in an extractor chamber. A volume of moist soil was poured loosely into the sleeve assembly to give a final sample that was 50–80 mm thick. Water was then poured on the plate to a depth of 1–2 mm. The water level was raised slowly (25 mm h^{-1}) until samples were under a positive head. The water level was then lowered slowly. Water was added and removed slowly to prevent displacement of fine soil particles in the sample matrix. Dispension of clays was not observed. Samples were allowed to drain overnight. Air pressure was applied to extract water and the samples were equilibrated at a soil-water potential of -100 kPa.

When the water-consolidated samples were at equilibrium, the cores were trimmed so that only the section 13–38 mm above the plate remained. Soil strength and bulk density measurements were then made as with the other series of cores. This bulk density is referred to as water-consolidated bulk density (WCBD). For a given soil, the range of bulk density was less than 0.05 Mg m^{-3} for the water-consolidated samples.

Probing

Soil strength measurements were made with a 5-mm diameter, flat-tip probe which was attached to a strain gage transducer or load cell. The complete assembly was moved vertically by a reversible electric motor. The motor was geared to operate at a constant penetration rate of 0.278 mm s^{-1} for 25 s.

Probe resistance was recorded on a chart as a continuous function of penetration depth. Values of probe resistance used in data analyses were taken from the charts at a depth of about 5 mm where the resistance had reached a constant value. Strength was calculated by dividing total probe resistance by the area of the probe. Three measurements were made on each face of the core, and the average was used in analysis. Although some test samples exhibited fracturing, they could be identified either visually or by observing the recorder tracings, and were excluded.

Calculations

Assuming that soil strength (S) is a continuous function of bulk density (ρ) and water content (θ), it can be approximated by a Taylor series of the form

$$S(\theta_0+\Delta\theta,\rho_0+\Delta\rho)=S_0(\theta_0,\rho_0)+\Delta\theta\frac{\partial S_0}{\partial\theta}+\Delta\rho\frac{\partial S_0}{\partial\rho}$$
$$+(\Delta\theta)^2\frac{\partial^2 S_0}{\partial\theta^2}+(\Delta\theta)(\Delta\rho)\frac{\partial^2 S_0}{\partial\theta\partial\rho}+(\Delta\rho)^2\frac{\partial^2 S_0}{\partial\rho^2} \quad (1)$$

However, if $\Delta S=S-S_0$, where $S_0=2$ MPa, $\Delta\theta$ (kg kg^{-1}) is the change in water content from current conditions to water content at 2 MPa (CRWC), and $\Delta\rho$ (Mg m^{-3}) is the change in bulk density from current conditions to bulk density at 2 MPa (CRBD), then the equation can be written as

$$\Delta S=A\Delta\theta+B\Delta\rho+C(\Delta\theta)^2+D(\Delta\rho)^2+E(\Delta\theta\cdot\Delta\rho) \quad (2)$$

where A, B,...E are constants. The series is truncated at the second derivative for ease of handling. Additional terms are assumed to be small and are ignored. Note that eqn. (2) has no intercept since $\Delta S=S-S_0$. Constants A–E are determined by regression of empirical values. Basically, eqn. (2) is a second order regression relating the difference in strength to differences in soil-water content and bulk density.

Soil strength, bulk density and water content values, as well as water contents and bulk densities at the 2 MPa soil strength level, had been determined by regression in Spivey et al. (1986). These were used here to develop A, B,...E for the group of 11 Ultisols tested. Bulk density was the major independent variable affecting strength. Although soils were tested at a water potential of −100 kPa, water content varied from 3 to 33%, and it was not always statistically insignificant.

To check the equations developed by calculating A–E, sections of data sets, samples equilibrated at −100 kPa potential, from both laboratory and field measurements (R.B. Campbell, personal communication, 1983) were used to calculate CRBD from eqn. (2). These were compared to empirical bulk density values at 2 MPa obtained from a regression of all data, including the 100 kPa samples used in eqn. (2).

In some cases, the critical rooting water content (CRWC) at 2 MPa of soil strength and -100 kPa of soil-water potential was required. It was estimated from either the average CRWC of the samples used to develop A–E of eqn. (2) or from a regression equation that linearly related CRBD and CRWC. This regression and eqn. (2) were solved numerically. They were iterated starting with eqn. (2) and an estimate of CRWC$=0$ to obtain an estimate of CRBD which was used in the linear regression to solve for a better estimate of CRWC. Iterations continued until there were no changes in the third place of accuracy.

RESULTS AND DISCUSSION

Coefficients of eqn. (2) were determined from data that contained 426 sets of readings for the 21 soils described earlier (Spivey et al., 1986). These soils were generally Ultisols with sandy to sandy loam textures. Bulk densities of the samples ranged from 0.78 to 1.86 Mg m^{-3}; CRBD ranged from 1.25 to 1.76 Mg m^{-3} for the different soil types. Although all samples were brought to equilibrium at -100 kPa soil-water potential, water content varied from 3 to 33% on a weight basis; CRWC[1] ranged from about 0 to 29%. Despite the wide range, water content was not a significant factor in eqn. (2) for some cases studied.

For example, when the whole 426-member data set was used in the regression, the third term of eqn. (2) was not significant and there was no significant difference between using all of the other terms ($R^2=0.87^{**}$) or just the bulk density terms (2nd and 4th terms; $R^2=0.86^{**}$). These resulted in the following equations

$$\Delta S = 8.42\,\Delta\rho + 10.7\,(\Delta\rho)^2 \tag{3}$$

$$\Delta S = -3.19\times10^{-3}\,\Delta\theta + 8.14\Delta\rho + 9.12\,(\Delta\rho)^2 - 2.52\,\Delta\rho\cdot\Delta\theta \tag{3a}$$

Similarly, when the data were limited to CRBD ± 0.3 Mg m^{-3}, a reasonable limitation for a Taylor series expansion, only the two bulk density terms were significant ($R^2=0.86^{**}$). This resulted in

$$\Delta S = 8.75\,\Delta\rho + 15.1\,(\Delta\rho)^2 \tag{4}$$

Graphic examples of these are shown in Fig. 1.

Regression equations were tested against both laboratory and field samples of Norfolk soil from separate studies. Critical rooting bulk density was calcu-

[1]Conditions at 2 MPa were determined by regression of bulk densities or water contents vs. strength for each soil (Spivey et al., 1986).

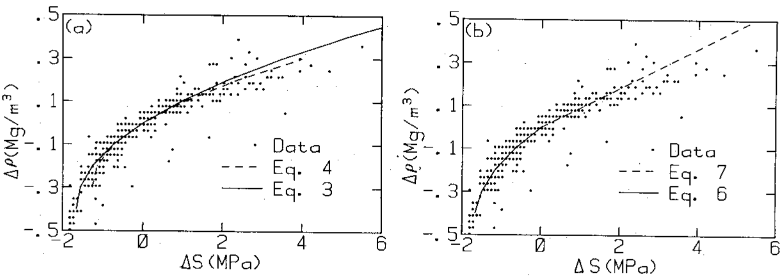


Fig. 1. Changes of bulk density ($\Delta\rho$) and strength (ΔS) plotted for (a) the data and eqns. (3) and (4), and (b) the data and eqns. (6) and (7).

lated from eqns. (3), (3a) and (4). Only a fraction of samples were equilibrated at -100 kPa soil-water potential and used to calculate the values in Table I. These closely approximated the values which were obtained by regression of all of the data in the same manner as in Spivey et al. (1986).

As mentioned above, in most cases water content could be ignored and the two bulk density terms of eqn. (2) could be used to calculate CRBD. However, in some cases shown in Table I water content was used. Here CRWC was assumed to be zero or was estimated from the mean value of CRWC values measured from the 31 soils (0.0997) or estimated from a linear regression of CRWC and CRBD. The latter two values exhibited similar and accurate results. The estimation method iterated the solution of eqn. (3a) by updating the value of CRWC in the linear regression equation

$$\mathrm{CRWC} = -0.537\ \mathrm{CRBD} + 0.9387 \tag{5}$$

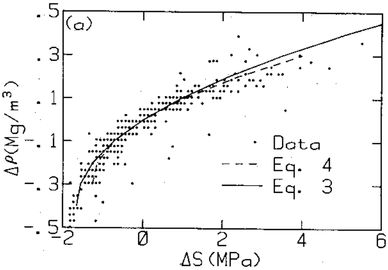
which was obtained by a regression of the 31 original soils sampled

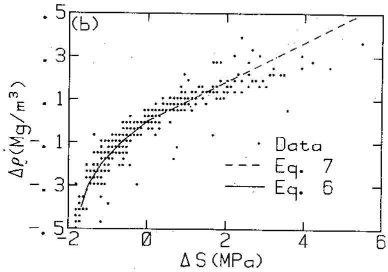
TABLE I

Measured and calculated critical rooting bulk density (CRBD) values for the laboratory and field samples

	Field samples			Laboratory samples
	Horizon A	Horizon E	Horizon B	Horizon A
Measured bulk density (Mg m^{-3})				
Sample size	43	54	20	160
Mean	1.61	1.63	1.49	1.65
Calculated bulk density (Mg m^{-3})				
Sample size	3	7	4	16
Eqn. (3)	1.62 (0.07)[1]	1.57 (0.05)	1.47 (0.08)	1.63 (0.03)
Eqn. (5)	1.61 (0.06)	1.56 (0.05)	1.47 (0.08)	1.63 (0.02)
Eqn. (4) (CRW=0)[2]	1.61 (0.07)	1.51 (0.06)	1.37 (0.13)	1.63 (0.03)
Eqn. (4) (CRWC=0.0997)	1.62 (0.07)	1.58 (0.05)	1.45 (0.11)	1.63 (0.03)
Eqn. (4) (iterated)	1.61 (0.06)	1.58 (0.04)	1.48 (0.09)	1.63 (0.03)

[1]Numbers in parentheses are standard deviations.
[2]CRWC is critical rooting water content.





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	Horizon A	Horizon E	Horizon B	Horizon A
Measured bulk density (Mg m^{-3})				
Sample size	43	54	20	160
Mean	1.61	1.63	1.49	1.65
Calculated bulk density (Mg m^{-3})				
Sample size	3	7	4	16
Eqn. (3)	1.62 (0.07)[1]	1.57 (0.05)	1.47 (0.08)	1.63 (0.03)
Eqn. (5)	1.61 (0.06)	1.56 (0.05)	1.47 (0.08)	1.63 (0.02)
Eqn. (4) (CRW=0)[2]	1.61 (0.07)	1.51 (0.06)	1.37 (0.13)	1.63 (0.03)
Eqn. (4) (CRWC=0.0997)	1.62 (0.07)	1.58 (0.05)	1.45 (0.11)	1.63 (0.03)
Eqn. (4) (iterated)	1.61 (0.06)	1.58 (0.04)	1.48 (0.09)	1.63 (0.03)

[1]Numbers in parentheses are standard deviations.
[2]CRWC is critical rooting water content.

TABLE II

Measured and calculated critical rooting bulk density (CRBD) values for the laboratory and field samples of Norfolk soil using the equations for strengths less than or greater than 2 MPa

	Field samples			Laboratory samples
	Horizon A	Horizon E	Horizon B	Horizon A
Measured bulk density ($Mg\ m^{-3}$)				
Sample size	43	54	20	160
Mean	1.61	1.63	1.49	1.65
Calculated bulk density ($Mg\ m^{-3}$) $\Delta S<0$				
Sample size	3		3	8
Eqn. (6)	1.63 (0.07)[1]	-	1.46 (0.09)	1.65 (0.03)
Eqn. (6a) (iterated)	1.61 (0.06)	-	1.54 (0.03)	1.65 (0.04)
Calculated bulk density ($Mg\ m^{-3}$) $\Delta S>0$				
Sample size		7		8
Eqn. (7)	-	1.56 (0.06)	-	1.63 (0.01)
Eqn. (7a) (iterated)	-	1.60 (0.03)	-	1.63 (0.01)

[1]Numbers in parentheses are standard deviations.

($R^2=0.67^{**}$). The case where CRWC = 0.0997 probably gave good results, since the soils used for developing eqn. (3a) and for testing were similar in texture and physiographic province. It exhibits at least a limited range of application for the method.

Data were also analyzed through eqn. (2) by splitting samples into cores with strength greater than 2 MPa (1974 samples) and those with strength less than 2 MPa (250 samples). For the former, $\Delta\rho$,$\Delta\theta$ and $(\Delta\theta)^2$ terms were significant in eqn. (2), giving $R^2=0.84^{**}$. If $\Delta\rho$ is used alone, it gives $R^2=0.82^{**}$ which is not significantly different. For the samples of less than 2 MPa strength, terms $\Delta\rho$, $(\Delta\rho)^2$, $(\Delta\theta)^2$, $(\Delta\rho)$ and $(\Delta\theta)$ were significant, giving $R^2=0.94^{**}$. However, this was not different from the case where only $\Delta\rho$ and $(\Delta\rho)^2$ were used, also giving $R^2=0.94^{**}$. If $\Delta\rho$ only is used, as in the previous case, $R^2=0.82^{**}$, which is significantly different from 0.94. For $\Delta S<0$, these resulted in

$$\Delta S=7.52\,\Delta\rho+8.38\,(\Delta\rho)^2 \tag{6}$$

$$\Delta S=7.41\,\Delta\rho+7.66\,(\Delta\rho)^2-29.8\,(\Delta\theta)^2-13.4\,\Delta\rho\cdot\Delta\theta \tag{6a}$$

and for $\Delta S>0$

$$\Delta S=10.9\,\Delta\rho \tag{7}$$

$$\Delta S=9.89\,\Delta\rho-2.38\,(\Delta\theta)+334\,(\Delta\theta)^2 \tag{7a}$$

Results of laboratory and field samples calculated from these regression equations are shown in Table II. Despite the fact that this case breaks the

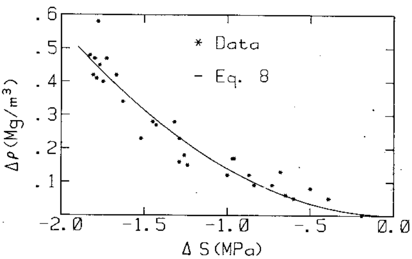

Fig. 2. Changes of bulk density ($\Delta\rho$) and strength (ΔS) between critical rooting and water-consolidated conditions plotted for the data and the regression of eqn. (8).

regression down into two formulae and the R^2 values are slightly better, the calculated results from the field samples (Table II) are not improved over Table I.

Values from the samples were also compared to the growth-limiting bulk densities (GLBD) from Daddo and Warrington (1983). Their GLBDs for a Norfolk loamy sand are calculated from texture as 1.78, 1.76 and 1.58 Mg m^{-3} for the Ap, E and Bt horizons, respectively. Growth-limiting bulk density is calculated at -33 kPa soil-water potential, and CRBD is calculated at -100 kPa soil-water potential. A regression comparing the two (Spivey et al., 1986) was used to calculate the CRBD from GLBD for the three horizons as 1.63, 1.62 and 1.50 Mg m^{-3}, which compare well with the measured or calculated values shown in Tables I and II.

Finally, CRBD was related to water-consolidated bulk density (WCBD) where loose samples are saturated and permitted to dry to -100 kPa soil-water potential under controlled conditions (Spivey et al., 1986). A significant relationship was found in a linear regression between CRBD and WCBD ($R^2=0.56^{**}$, $n=33$). Statistical significance was greater ($R^2=0.99^{**}$ for $n=33$) when the square root of the change in bulk density between critical rooting and water-consolidated bulk densities (CRBD–WCBD) was related to the change in soil strength between the two conditions (strength at WCBD $-$ 2 MPa), giving

$$(\Delta\rho)^{0.5}=0.140\,S_{WCBD} \tag{8}$$

where S_{WCBD} is the strength measured at WCBD. This is graphically illustrated in Fig. 2. This was not compared with field data separate from the original samples. However, the technique shows promise as another way to determine CRBD from a simple test.

For wider application than Coastal Plain soils, the formulae would have to be tested over a data set with a larger range of texture, structure and water content. However, the regression, derived from the disturbed laboratory samples, applies well to the field samples tested. Furthermore, it is possible that parallel slopes of strength vs. bulk density curves for a specific water content (Saini et al., 1984) negate the need to use water content to estimate CRBD in this manner.

CONCLUSIONS

Regression equations were developed that related the change in soil strength associated with the change in bulk density and water content between some measured value and the critical rooting conditions for soils equilibrated at -100 kPa soil-water potential.

Relationships among these changes were simplified in many cases by the elimination or estimation of water content at a soil strength of 2 MPa and soil-water potential at -100 kPa. Calculated CRBD agreed closely with experimental values. This may be because all soils used were similar in texture and physiographic origin. Nevertheless, it did make the calculations easier and demonstrated at least a limited range of usefulness for the regressions with and without the inclusion of changes in water content.

Of course, inclusion of more soil types in the analysis would improve the accuracy and applicability of the equations. A method of estimating the water content at 2 MPa soil strength and -100 MPa soil-water potential from easily-measured or calculated soil parameters such as CRDB or texture would be useful (Gupta and Larson, 1979).

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